is dependent upon the step response function having a maximum point for some τ less than infinity. That this function has a maximum point has been shown to be physically reasonable. Quantitatively, the stall flutter velocities are highly dependent upon the exact shape of the step response function. Continued theoretical and experimental studies are required to confirm this theory and to provide usable data in order to predict the critical velocity of a given airfoil at a given angle of incidence.

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Use of Accelerometers to Determine Tilts and Displacements

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Introduction

THIS paper has grown from the authors' interest in seeing whether horizontal accelerations during earthquakes could be winnowed from accelerometer data which includes response both to horizontal accelerations and to tilts about a horizontal axis. (We will use the word horizontal to imply a direction in inertial space, i.e., a normal to the long-term average gravity vector.) On all horizontal accelerometers, tilts in a gravity field produce a spurious output equal to $g \sin \Theta$ which is indistinguishable from horizontal accelerations. Since available accelerometers and pendulums have better threshold sensitivity for tilt than gyros, it is of interest to see whether simultaneous

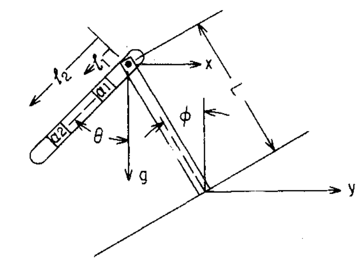

Fig. 1 Schematic of two accelerometers mounted on a pendulum.

angular and linear motions can be determined without using gyros.

The investigation has interest in such diverse areas as inertial guidance, gyro and accelerometer test platforms, and motion of oceanographic platforms such as FLIP (Floating Instrument Platform). Sem and Lear[1] have summarized instruments for determining stability of platforms. They refer to a proposal by Tsutsumi for determining translational and rotational motion over the frequency band 0–10 Hz using two pendulums. Bradner[2] has proposed using two accelerometers.

This paper discusses some of the pendulum and accelerometer configurations that we have examined. We show that all such systems require fourth-order time integrals to determine horizontal displacement unless the natural period of one of the sensors is much longer than the passband of interest.

Instruments Measuring Horizontal Motion

We will consider the case of small horizontal translations that are orthogonal to the horizontal axis of small rotary motions and have comparable frequency spectrum. The notation convention is shown in Fig. 1. ψ is the only measurable parameter. The equation of motion, in Laplace transform notation, using small angle approximation is

$$(Is^2+\mu as+mag)\Psi = mas^2Y-[(I+maL)s^2-mag]\Phi \quad (1)$$

where a is the distance from the pivot to the pendulum's c.m. Since the coefficients of s^2Y and $g\Phi$ are identical, two identical pendulums set side by side will not aid in separating these two inputs. The displacement Y cannot be found by double integration unless a method can be found to measure or eliminate Φ.

Consider first two ideal accelerometers mounted on a pendulum, as proposed by Bradner.[2] (See Fig. 1.) Accelerometer outputs α_1 and α_2, can be written without reference to the equations of motion of the pendulum,

$$\alpha_i = -l_i\ddot{\theta} - g\theta + \ddot{x}; \quad i = 1, 2 \quad (2)$$

Solving the two equations for θ and then X yields

$$\theta = (\alpha_1-\alpha_2)/s^2(l_1-l_2) \quad (3)$$

and

$$X = \alpha_1/s^2 + l_1(\alpha_1-\alpha_2)/s^2(l_1-l_2) + g(\alpha_1-\alpha_2)/s^4(l_1-l_2) \quad (4)$$

Fourth-order integration of accelerometer outputs is thus required to obtain horizontal displacement.

Note that no reference is made to the angular tilt of the frame (ϕ) in Eq. (2). This is due to a fortuitous choice of reference at the pivot. If one is interested in the time history of actual "ground" motion ("y," using the input conventions of Fig. 1) one finds x contaminated by and inseparable from ϕ, and, since

$$y = x + L\phi \quad (5)$$

the variable y cannot be found. One can always bypass this

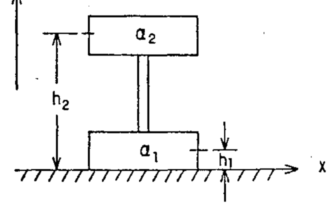

Fig. 2 Schematic of two accelerometers mounted on stand at heights h_1 and h_2.

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difficulty by mounting the pendulum in some manner so that ϕ induces no horizontal motion at the pivot, e.g., by placing the pivot at actual ground level where $x = y$.

This difficulty can be circumvented. Consider two accelerometers mounted rigidly at different levels, h, on a stand, as shown in Fig. 2. This is a limiting case of the instrument previously considered, letting the damping constant of the pendulum become infinite.

Take the reference axis of tilt to be at the base of the stand. The outputs are

$$\begin{aligned} \alpha_1 &= s^2X+(s^2h_1+g)\Phi \\ \alpha_2 &= s^2X+(s^2h_2+g)\Phi \end{aligned} \tag{6}$$

Hence,

$$X = \alpha_1/s^2-(s^2h_1+g)(\alpha_1-\alpha_2)/s^4(h_1-h_2) \tag{7}$$

$$\phi = (1/s^2)(\alpha_1-\alpha_2/h_1-h_2) \tag{8}$$

Displacement can therefore be determined only by fourth-order integration. If we had two pendulums with different moments of inertia, damping, length, etc. we would still have found that fourth-order integration is necessary for determining horizontal displacement unless terms in the equations can be neglected.

The fourth-order equation that appears in all the systems can be avoided if a sensor can be made to have $g\phi$ term negligibly small compared with the $\dot{\phi}$ and $\ddot{\phi}$ terms throughout the passband of interest. A long-period horizontal pendulum has the desired property, since $I\omega^2 \gg mag$, and the general pendulum equation (2) becomes approximately

$$(I_r s^2+\mu_r a_r s)\Psi_r \doteq I_r s^2\Phi \tag{9}$$

where we have used the subscript r as a reminder that the long-period pendulum acts somewhat as an inertial reference during the observations. Note that a gyro inertial reference can be considered in this context as simply a pendulum with very long period because of its large I and small a.

A short-period pendulum and a long-period pendulum mounted on the same base will have equations of motion (1) and (15) which can be combined to give

$$maI_r s^3X \doteq (Is^2+mag)(I_r s+a_r\mu_r)\Psi_r-(I_r s)(Is^2+a\mu s+mag)\Psi \tag{10}$$

This expression involves integrals up to the third order. If the damping of the long-period pendulum is made small enough so that the term in μ can be ignored, then X can be found by first- and second-order integrations.

The tilt amplitude in microseism disturbances is generally about the same magnitude as the normalized horizontal acceleration, $\ddot{x}/g$, and has a frequency spectrum of comparable shape. Therefore the fractional error from neglecting the terms in the above equations will be the order of

$$(m_r a_r g+a_r\mu_r\omega)/I_r\omega^2$$

or in terms of damping ratio ζ_r, undamped pendulum angular frequency ω_{ro} and microseism frequency ω, the fractional error will be the order of

$$({\omega_{ro}}^2+2\zeta_r\omega_{ro}\omega)/\omega^2$$

Summary and Conclusions

We have presented a number of schemes designed to measure time-varying horizontal motion in the presence of random tilts, without the use of gyros. A quite simple class of instruments is employed as a basis.

For inertial guidance purposes, where real time outputs are required, these methods are probably not suitable in their present form, due to possible magnification of errors upon multiple integration. For test pad monitoring where output is not immediately required, these instruments, which are relatively easy to set up, should be useful. Use of accelerometers with $10^{-8}g$ sensitivity in the appropriate low-noise passband, would provide information on the micromotion of test pads and stable platforms previously unattainable.

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Finite Deflection of a Shallow Spherical Membrane

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Introduction

THE present Note is concerned with moderately large axisymmetric deformation of a linearly elastic shallow spherical membrane loaded by a uniform internal pressure and clamped against deflection at its outer edge. Nonlinear membrane problems of this type are of interest because membranelike structures find several applications in aerospace technology (see Rossettos[1]) and because they provide relatively simple examples of boundary-layer behavior in nonlinear shell theory. The particular problem under consideration was first discussed by Bromberg and Stoker.[2] They found a power series solution and also pointed out that boundary-layer behavior occurred for small loads. They obtained an asymptotic solution which illustrated this effect. Because of the rapid changes in the dependent variables that occur in the boundary layer, this problem can provide a severe test for various numerical methods. It has been used as such by Goldberg[3] and Perrone and Kao.[4] The purpose of the present work is to report results which are thought to be more accurate than those of previous workers and in doing so to call attention to the importance of giving sufficient consideration to boundary-layer effects when finding a numerical solution to a problem of this type. The appropriate differential equations are solved numerically using a finite-difference method and for small values of load the results are verified by comparison with a three-term perturbation solution found by the method of matched asymptotic expansions (see Van Dyke[5]). It is hoped that these improved results will be of use to investigators who wish to use this problem as a test case to evaluate proposed numerical methods for solution of shell equations.

Governing Equations

Consider a uniformly pressurized (the magnitude of the pressure is p) shallow spherical membrane with radius of curvature R, subtended angle 2α, and thickness h made of a linearly elastic material with modulus of elasticity E and Poisson's ratio ν. The equations governing moderately large axisymmetric deflections of such a structure can be obtained from the work of Reissner,[6] to which the reader is referred for the details of the derivation. These are omitted from the present note for the sake of brevity. In the following discussion all variables and parameters appearing in the equations are understood to be dimensionless.

The basic differential equation can be found from Eq. (5.2) of Reissner's[6] paper to be

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